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EVALUATING AIR COMBAT MANEUVERING ENGAGEMENTS. Volume I.
Vol. I - Methodology.

CENTER FOR NAVAL ANALYSES
1401 Wilson Boulevard
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Operations Evaluation Group

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

aerial warfare, maneuverability, Markov processes, mathematical analysis, mathematical models, pilots, probability, ranges (facilities), simulation, stochastic processes, training

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This study describes two stochastic models useful in evaluating air-to-air engagements between high-performance fighter aircraft. The Maneuver Conversion Model is applicable to engagements where a successful outcome is determined primarily by maneuvering effectiveness of the combatants. The Firing Sequence Model is intended for analysis of engagements where a successful outcome depends primarily on aircrew ability to capitalize on weapon performance. Common measures of effectiveness, such as the probability of achieving first weapon firing and the exchange ratio, may be estimated by these models. (over)

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20. The methodology for both models is based on the theory of semi-Markov processes. Volume I presents the analytic methodologies for the models and provides illustrations with simulated data. Volume II contains an analysis of data gathered under CNO Project P/V2 (Battle Cry), and illustrates the Maneuver Conversion Model methodology.

Volume II is a classified volume, entitled "Results of Harrier Versus F-86 (U)."

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2. Following involvement with the analysis of the AV-8A Harrier thrust vectoring capability in 1971, CNA analysts have developed two stochastic models which may be useful in evaluating air-to-air engagements between high-performance fighter aircraft. These models use test range data to derive probabilities of win, loss, and draw as functions of engagement duration and are based on the theory of semi-Markov processes. The Markov assumption made in the analyses, i.e., that the future evolution of an engagement depends only on the present state and ignores the past, is not strictly true for air-to-air combat. However, range data suggests that the assumption may be a good approximation, especially for short engagements. The models are applicable for analysis of one-against-one and two-against-one engagements, but do not apply for analysis of more complex engagements. Furthermore, the models require range data inputs and are of very limited utility in inferring combat capability from test range capability.

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SUMMARY

INTRODUCTION

Air-to-air warfare capability is an integral part of the U.S. defense capability, and recent experience in both Southeast Asia and the Arab/Israeli conflicts reemphasizes the need for continual, comprehensive aircrew training in this type of combat. The U.S. has schools in which fighter pilots receive instruction in air combat maneuvering (ACM) and, more recently, instrumented ranges have been developed to expand our training capability. Regretfully, the analysis community has been slow in developing a unified methodology for evaluating a total system (aircraft, aircrew, weapon system, and tactics) during training. The main reasons for the difficulty are: (1) the complexity of air-to-air scenarios, (2) differences between training and actual combat, and (3) difficulties in reconstructing air-to-air engagements for analysis.

Some partial success has been achieved in analyzing ACM. The U.S. Air Force has used energy-maneuverability models successfully to design maneuver tactics based on optimal energy management. Such models generally cannot quantify less-than-optimal maneuvering, and do not lead to probabilities of win, loss, and draw. Attempts to use game-theory techniques have generally been unsuccessful although such techniques appear to have considerable potential.

Various ad hoc techniques have been used for analysis of ACM data obtained on test ranges, with emphasis being on statistical properties. Numerous useful measures of effectiveness (MOEs) have been formulated, but attempts to integrate the MOEs into an overall scheme which evaluates ACM effectiveness have been unsuccessful.

In 1971, AirTEvRon Four (VX-4) was tasked by CNO to evaluate the survivability of the AV-8A Harrier attack aircraft in a hostile fighter environment. Because of the Harrier's unique thrust-vectoring capability, the scope of the project was enlarged to include an ACM evaluation of the Harrier. At this time, no numerical methodology was available to quantitatively assess the value of the thrust vectoring in ACM. In an attempt to support with analysis the conclusions reached by the aircrews, CNA analysts and the VX-4 project officers developed an analytic evaluation scheme which has since become known as the Maneuver Conversion Model. Using this model, analysts were able to quantify the aircrew assessment of the value of the Harrier thrust vectoring for ACM. This early success stimulated research to extend the Maneuver Conversion Model and also to explore other ACM models.

This study describes the structure and numerical properties of two stochastic models of air-to-air combat that have proven useful in understanding the sequences of events observed in test-range engagements. These models (the Maneuver Conversion Model and the Firing Sequence Model) have important differences which will be discussed below. However, the models have several things in common:

- Both models are designed to use test-range data as inputs.
- A primary output for both models is the set of probabilities of win, loss, and draw as a function of engagement duration.
- The models do not attempt to evaluate specific tactics in a specific engagement; rather, they provide estimates of the probability of achieving a favorable tide of battle.
- Both models use semi-Markov analysis techniques and are based on a Markov assumption which approximately states that "the future evolution of an engagement depends solely on the present situation and not on what has gone before."

MANEUVER CONVERSION MODEL

The Maneuver Conversion Model results from thinking of the friendly fighter as being in one of the following states at any time:

• Win	Friendly fighter fires an effective weapon
• Offensive weapon	Friendly fighter fires a weapon
• Offensive	Friendly fighter has a significant advantage
• Neutral	Neither fighter has a significant advantage
• Defensive	Opponent has a significant advantage
• Defensive weapon	Opponent fires a weapon
• Loss	Opponent fires an effective weapon

Starting, for example, in the neutral state, the friendly fighter proceeds through some sequence of states until someone breaks off (a draw) or until someone wins. Range data is used to provide estimates of the probabilities of transition from one state to another. Also, the length of time spent in each state is random, and the range data is used to estimate the distribution functions of these times-in-state. With these estimates, the analyst can estimate the win, loss, and draw probabilities using semi-Markov techniques, providing he is willing to make the Markov assumption noted in the preceding subsection, i.e., that the future evolution of an engagement depends only on the present situation (present state and length of time in this state) and not on what has gone before. This assumption, however, cannot be strictly true since it would require that the pilots not tire and that they fight "without memory" (i.e., that they do not learn during the engagement). But range data suggests that the assumption is approximately true, especially for short engagements.

The definitions of the states depends on the types of aircraft and the tactics used. Typically, these definitions are in terms of relative speeds, relative positions, who has sight of whom, altitudes, and energies and are chosen to be mutually agreeable to the aircrews and analysts.

A primary output of the analysis is the set of probabilities of win, loss, and draw as functions of engagement time. Using these results, the exchange ratio may be calculated. Other outputs which can be gotten include:

- Probability of achieving first weapon firing.
- Expected fraction of time spent in any given state.
- Expected number of visits to any given state.
- Expected fraction of time spent in offensive or higher state.

In volume I, the methodology for this model is derived and illustrated using fictitious data; in volume II, the model is used to extend the results of the Harrier OpEval reported in reference 6.

The Manuever Conversion Model has been implemented at the Naval Air Test Center, Patuxent River, Md., the Naval Weapons Center, China Lake, Calif., the Air Combat Maneuvering Range, Yuma, Ariz., and the Center for Naval Analyses. The model has been used at the Naval Air Test Center for analysis of the F-11A thrust vector control system.

Research is continuing at ComFitAEWWingPac, the Naval Air Test Center, and the Center for Naval Analyses to extend the Maneuver Conversion Model to a continuum of states as opposed to the seven states listed above.

FIRING SEQUENCE MODEL

In simplest form, the Firing Sequence Model may be thought of as a condensed version of the Maneuver Conversion Model in which only two states, offensive weapon and defensive weapon, are used. For each engagement, the range data input to this model is the sequence of simulated firings, i.e., who shot at whom and at what time. With this data, the analyst can estimate the transition probabilities and time-in-state distributions, as he could for the Maneuver Conversion Model. The primary output from the analysis is the set of probabilities of win, loss, and draw as functions of engagement duration, and an advantage of this model is that these probabilities are gotten with a minimum of data collection and analysis. A disadvantage is that no information involving the other states is available.

This model can be easily extended to cover the situation that both combatants are in the offensive weapon state at the same time (e.g., in a head-on situation, both combatants may fire missiles at the same time). In principle, the Maneuver Conversion Model can be used in this situation; in practice, however, the numerical complexities are nearly prohibitive.

In volume I, the methodology for this model is derived and illustrated with fictitious data.

APPLICATION OF THE MODELS

These models have potential application in several areas. An important concern is the contribution that these models can make in extrapolating combat ACM capability from peacetime ACM capability. At present, these models require range data inputs (or the hypothesized equivalent) so that a primary requirement for such extrapolation is test-range data involving engagements between friendly fighters and threat aircraft in which "real-world" tactics are used. Even with such data available, extrapolation to combat situations would require great care.

In a peacetime setting, the models may be useful in evaluating a controlled flyoff of competing aircraft to assist in decision-making about future procurement. Also, the models may be used by the Fighter Weapons School to monitor the proficiency and improvement of students as they progress through training.

LIMITATIONS OF THE MODELS

Both models are designed primarily for analysis of the classical one-against-one engagement. Although some success has been achieved in studying the two-against-one engagement with these techniques, these models do not appear to be directly applicable to more complex, multiaircraft engagements. This study methodology should be regarded as a first step toward understanding the more complex engagements.

The data collected at instrumented test ranges is a primary input for these models. ACM engagements at test ranges are conducted under a variety of constraints which detract from the "realism" of the engagements. As a result, the models in this study must be used with great care if it is desired to use the test range results to infer ACM capability in any other setting.

The study methodology is based on the Markov assumption that the future evolution of the engagement depends only on the present situation and not on what has gone before. As noted above, this assumption is not strictly true although data suggests that it holds approximately, especially for short engagements. Research is continuing to develop methodology which incorporates some of the past history into the future evolution of the engagement.

INTRODUCTION

Although an air-to-air warfare capability has for years been an integral part of the U.S. defense posture, very few quantitative estimates of effectiveness have been available. This lack of information is due to three primary causes: (1) the complexity and variety of air-to-air scenarios, (2) differences between training operations and live combat operations, and (3) technological difficulties associated with the recording and reconstruction of events in air-to-air engagements. The first two impediments can be overcome with mathematical models. It is necessary to define probabilistic structures within which scenarios may be defined, variables quantified, interactions studied, and simulated weapon firings treated as live firings. With the introduction of instrumented training ranges, the data-collection problem has been solved and the way is clear for a systematic investigation of air combat.

Air-to-air combat analysis at CNA began in 1971. At that time, AirTEvRon Four (VX-4) was tasked by CNO to evaluate the survivability of the AV-8A Harrier attack aircraft in a hostile fighter environment. Because the Harrier was equipped with thrust vectoring--a technological advance with potential applicability to fighter warfare--the scope of the project was enlarged to include an evaluation of the usefulness of thrust vectoring in maneuvering (i.e., classic) air combat.

When the Harrier project was planned, no numerical techniques were available to assess the value of the thrust vectoring quantitatively. Consequently, the aircrews intended to rely on their subjective evaluations. In an attempt to support with analysis the conclusions reached by the aircrews, CNA analysts and the VX-4 project officers developed an analytic evaluation scheme that has since become known as the Maneuver Conversion Model. Using this model, analysts were able to quantify the aircrew assessment of the value of thrust vectoring for fighter application. This early success stimulated research to analytically exploit the mathematical structure of the Maneuver Conversion Model and also the exploration of other models of air combat.

This study describes the structure and numerical properties of two stochastic models of air-to-air combat that have proven useful in understanding the sequences of events observed in test range engagements. The models (the Maneuver Conversion Model and the Firing Sequence Model) are designed to extract information from the events observed in a sample of test range engagements. The methods not only provide a technique for estimating the probabilities of a win, loss, or draw, but also provide insight into aircrew maneuvering and weapon-firing performance.

The models presented in this study do not attempt to evaluate individual maneuvers or procedures used by aircrews to accomplish a kill in a specific engagement. Rather, the models seek out repetitions or patterns in the tides of battle and estimate the probability of achieving a favorable tide of battle. In this sense, these models differ conceptually from the more common ACM evaluation techniques summarized below.

The inherent complexity of air-to-air combat negates the use of a single model to describe all possible air combat scenarios. The two models presented in this study are therefore limited in applicability. Both are designed to model classical one-against-one air combat maneuvering (ACM) engagements and do not model more complex, multi-aircraft engagements. The Maneuver Conversion Model assumes that at least one of the combatants is required to maneuver to the opponent's rear hemisphere before weapons may be employed, and that the ability of the aircraft to make such maneuvers is quantified by this model. Other outputs of the model are the probabilities of win, loss, and draw as a function of time. The Firing Sequence Model, however, makes no such assumption and provides no direct quantification of maneuvering ability. Rather, it models the sequences of weapon firings that an engagement may generate but without regard for the details of maneuvering or weapon use. Its primary outputs are the probabilities of win, loss, and draw as functions of time. Research into refinements of these models as well as research into models applicable to more complex engagements is continuing. The models in this study should be regarded as a first step toward understanding these more complex engagements.

Volume I of the study begins with a brief historical overview. The overview is divided into two parts, the first being a description of the evolution of air-to-air combat from its beginnings in World War I, and the second being a summary of evaluation techniques that have been applied by combat analysts. The historical background is followed by a discussion of each of the models - the Maneuver Conversion Model and the Firing Sequence Model. For each model, the assumptions, analysis methodology, and numerical results are described. The model descriptions are followed by a section describing the data-collection requirements when the models are used to evaluate trial engagements conducted on an instrumented range. The first volume concludes with a brief discussion of possible areas of application. Detailed mathematical developments are contained in appendixes A and B.

Volume II is limited to an extension of the discussion given in reference 6 for the AV-8A Harrier evaluation. The numerical results further illustrate the use of the Maneuver Conversion Model.

HISTORICAL BACKGROUND

EVOLUTION OF AIR COMBAT

Air-to-air combat had its origin in World War I. It resulted from an attempt to counter the use of aircraft for tactical aerial reconnaissance. The early fighters were two-place aircraft armed with a machine gun mounted in the rear seat. The initial employment tactics consisted of flying in front of the unarmed reconnaissance plane and firing toward the rear. In an attempt to counter this tactic, the Germans developed a forward firing machine gun to be mounted on the reconnaissance aircraft. Shortly thereafter, allied reconnaissance aircraft were similarly armed. This technological advance not only changed the defensive capability of the reconnaissance aircraft, but actually gave birth to what is known as classical air combat maneuvering (ACM).

The introduction of the forward-firing weapon drastically altered the nature of the aerial combat. Rather than flying in front of the opponent and firing to the rear, pilots attempted to achieve a position in the opponent's rear hemisphere before firing. As the opponent was rarely cooperative, highly dynamic pursuit/evader maneuvering ensued. Aircrews very appropriately named such an engagement a "dogfight." Very early in the evolution of this dynamic air combat, referred to as ACM, fighter aircrews recognized the need to be cautious because of the fighter's vulnerability to attack by other opposition aircraft while concentrating on a single opponent. This caution resulted in the development of various flight formations and engagement tactics designed to provide an aggressive (kill) capability without compromising the required lookout for other enemy fighters. It is interesting to note that the tactical principles that balanced aggression and caution in the early days of ACM have remained unchanged in both character and importance to this day.

Throughout World War II and the Korean conflict, the nature of ACM remained essentially unchanged. More sophisticated gun systems and more powerful aircraft were introduced, but the basic maneuvering characteristics of the ACM engagement remained. Throughout these conflicts, aircrews adjusted engagement tactics to fit changing tactical situations and continually relearned the well-known tactical lesson -- you never see the one that kills you.

World War II did generate a new fighter mission -- the fighter interceptor. In this role, the high-speed, well-armed fighter aircraft were vectored to intercept and engage the heavy bomber as it proceeded to its target. Engagements of this type were generally characterized by high-speed fighter firing passes with some evasive maneuvering to avoid the bomber gun defenses.

In the mid-1950s, the air-to-air guided missile appeared in the fighter armament inventory. Although both radar-guided and heat-seeking varieties were available, the heat-seeking missile represented the technological breakthrough which most strongly affected

the ACM engagement. Although the radar-guided missile was conceptually an all-aspect weapon, the difficulty in maintaining radar track on a maneuvering opponent greatly diminished the effectiveness of this missile in the classical ACM role, and the radar-guided missile was relegated to use in the interceptor role.

The heat-seeking missile, on the other hand, had the very desirable characteristic of pursuing the target after launch, independent of postlaunch maneuvers of the firing aircraft. This significantly decreased the vulnerability of the firing aircraft during the weapons-firing portion of the engagement. As with gun systems, the early heat-seeking missiles were effective only when fired in the opponent's rear hemisphere. Again, the major technological advance failed to alter the dynamic maneuvering nature of ACM.

By the early 1960s, the role of the pure fighter had essentially been replaced by the dual-mission fighter interceptor. Armed with both radar and infrared (heat-seeking) guided missiles, the fighter interceptor was prepared for either mission. This dichotomy--radar-guided weapons for head-on, closing shots and heat-seeking missiles and guns for the ACM engagements--continued into the 1970s.

In these latter years, widespread use of the digital computer and the development of usable rapid lock-on modes of radar operation significantly increased the usefulness of radar-guided missiles in ACM engagements. Simultaneously, breakthroughs in seeker sensitivity resulted in heat-seeking missiles with some capability in an opponent's forward hemisphere. Current fighter interceptor aircraft are armed with a mix of weapons, each type useful in either kind of engagement.

As the pure fighter became subsumed by the dual-mission fighter interceptor, aircrews trained for perfection in both roles. The training engagement generally consisted of a forward quarter simulated intercept during which weapons would be employed, followed by an ACM engagement after the initial pass. During each portion of the engagement, simulated missiles are launched and defensive maneuvers evaluated. In the interceptor role, success can be equated with the ability to set-up and launch forward-hemisphere weapons, counter opponent firings, and prepare for ACM. In the ACM role, success is equated with maneuvering effectiveness, i.e., the fighter crew must maintain a clear picture of the constantly changing tactical situation, outmaneuver the opponent, obtain a position in the opponent's rear hemisphere, and fire weapons. The development of methods to evaluate the total system (aircraft, aircrew, weapon system, and tactics) in these training flights has been a goal that has eluded combat analysts for the entire history of aerial combat.

While the underlying principles have remained essentially unchanged, the enlargement of the basic fighter mission, the increase in aircrew workload, and the demands of coordination and timing have drastically altered air combat training requirements. Experience in both Southeast Asia and the Arab/Israeli conflicts, reemphasized the need for continual, comprehensive aircrew training. These training demands stimulated the development of

instrumented ranges for the required routine air combat training. Developed on the principle that "seeing is believing," these ranges permit aircrews to review training engagements to isolate mistakes, evaluate weapon firings, and discuss tactical achievements and deficiencies. The training potential of these ranges will take years to exploit. The models in this study are designed to contribute to this exploitation.

AIR COMBAT EVALUATION TECHNIQUES

The development of methodologies for evaluating air-to-air engagement has proceeded along two lines. One approach has been to analyze the maneuver dynamics of two engaging aircraft. Such models are based on physical principles and have been used to improve airframe design, drive manned simulations, and assist in postulating engagement tactics. The second modeling approach is event oriented and consists of formulating simple measures of effectiveness of the various elements (aircraft, weapon system, tactic, etc.) of ACM engagements, and computing these measures with the data gathered at a test range.

Energy-Maneuverability Models

These models are used to analyze the maneuver dynamics and have been the most successful for ACM analysis. The basic ideas of energy maneuverability (references 1 and 2) have been extended using variational and differential gaming techniques to determine optimal maneuver tactics. This methodology uses the physical equations of motion to relate the effects of accelerated flight on system energy. It has been used to design maneuver tactics based on energy management and to identify flight regimes where fighter airframe performance exceeds that of a postulated opponent. The methodology also helps a pilot understand the limits and potential performance of opposing aircraft. With this methodology, methods of efficient energy management may be derived which maximize the number of maneuver options within the limits of the airframe. This use of energy-maneuverability theory is called the maximum maneuver concept and results in the tactic of engaging enemy aircraft only at altitudes and airspeeds for which maneuver options are favorable.

Variations of the energy-maneuverability models have been successfully used in "man-in-the-loop," real-time ACM simulations. These simulators are adaptations of the differential equations governing motions of powered vehicles in three-dimensional space in response to actions by the pilots. Consequently, these simulations are more realistic than representations of engagements based only on analytic energy-maneuverability models.

The energy-maneuverability models generally cannot be used to quantify either the effect of less-than-optimal maneuvering or the use of weapon systems to compensate for the less-than-optimal maneuvering. These deficiencies are somewhat overcome in the man-in-the-loop simulators. In particular, the TACTICS II simulation (reference 3) is an effort to incorporate weapons performance into the analysis.

Game-Theory Models

Attempts to use game-theory techniques for analysis of ACM models have generally been unsuccessful. Some limited success has been achieved in modeling pursuit/evader situations with differential gaming techniques (reference 4). To date, such models have been deterministic and highly idealized. However, these techniques appear to offer the potential of a detailed understanding of ACM models.

Test-Range Models

These model ACM in terms of sequences of events as observed on test ranges; emphasis is on statistical properties rather than physical processes. Despite the formulations of numerous measures of effectiveness, attempts to integrate them into an overall scheme which evaluates ACM effectiveness have been unsuccessful. The present report addresses this problem.

STOCHASTIC MODELS

The models in this study are designed to estimate relative effectiveness in one-on-one maneuvering combat. The opposing forces are each assumed to consist of a specific family of pilots, flying a fixed aircraft type, having a specific weapon system, and using a predefined family of tactics. The estimation is based on a sample of air-to-air engagements conducted at an instrumented range. In each sample engagement, all missile firings are assumed to fail and the engagement allowed to continue until termination for tactical reasons by either combatant. The analytic methodologies described in this study are designed to use the sample engagements to estimate the probabilities of a win, loss, and draw. Additionally, the methods provide estimates of several tactical measures of effectiveness (MOEs) that provide insight into the maneuvering and weapon-system employment characteristics of the engaging forces.

MANEUVER CONVERSION MODEL

The Maneuver Conversion Model results from structuring air-to-air engagements as semi-Markov processes with absorbing states corresponding to the possible outcomes of the engagements. The transitions between states correspond to observable events that occur during engagements; transition probabilities and parameters for time-in-state distributions can be estimated from test-range data. The output of the model consists of the probabilities of the possible outcomes of the engagement and other tactical MOEs related to the different transitions and states. The advantage of this approach is that the states can be defined to be operationally meaningful and can account for the interaction of hardware performance and human factors.

The semi-Markov processes may be analyzed by Monte Carlo simulation or by probabilistic methods using semi-Markov theory. Both methods are practical and useful; in this report, attention is restricted to the probabilistic methods.

The methodology developed in this section represents continued analytic development of the Maneuver Conversion Model first introduced in reference 5. The methodology is implemented at the Naval Air Test Center, Patuxent River, Md., the Naval Weapons Center, China Lake, Calif., the Air Combat Maneuvering Range (ACMR), Yuma, Ariz., and the Center for Naval Analyses.

Assumptions

Specific assumptions used in constructing the semi-Markov model are:

- Combatants may be described as being in one of seven engagement states (table 2) based on prescribed rules for the particular combatants and type of engagement.[1]

[1] Research is ongoing at ComFitAEWWingPac, the Naval Air Test Center, and the Center for Naval Analyses to extend the maneuver to a continuum of states.

- The Markov assumption: The probability of transition from the current state to the subsequent state by either combatant is independent of the prior engagement states and time spent in those states and is dependent only on the current and subsequent states. The transition probabilities do not change with time and are assumed to be known (usually estimated from range data).
- For each engagement state, S, the time-in-state, t_S, is a random variable with cumulative probability distribution F_S . These distributions do not change with engagement time and are assumed to be known (usually estimated from range data).
- Instantaneous passage through one or more states is not permitted; a non-zero (but perhaps small) time-in-state is required. Expressed mathematically, $F_S(0) = 0$ for all S.
- Simultaneous firing opportunities for both combatants is prohibited.

TABLE 2

ENGAGEMENT STATES

(Relative to the Evaluation Fighter)

Engagement state	Abbreviation	Definition
Win	W	Evaluation fighter wins (fires an effective weapon).
Offensive weapon	OW	Evaluation fighter fires a weapon.
Offensive	O	Evaluation fighter has a significant tactical advantage.
Neutral	N	No combatant has a significant tactical advantage.
Defensive	D	Opponent has a significant tactical advantage.
Defensive weapon	DW	Opponent fires a weapon.
Loss	L	Opponent wins (fires an effective weapon).

The first three assumptions permit a semi-Markov process analysis of the engagement. The remaining two assumptions are not strictly necessary from the mathematical viewpoint, but they simplify the analysis greatly.

The idea of expressing a "dogfight" as a collection of states is perhaps novel; however, informal interviews with many pilots suggests that such thinking is not altogether foreign to them. Formally, the analyst must define the states explicitly in terms of speeds, ranges, angles, etc., as appropriate to the aircraft and weapon systems in use.

The second and third assumptions are consistent with ACM data as reported in reference 6, although the Markov assumption cannot be completely valid. It would require that the pilots not tire and that they fight "without memory," i.e., that they do not learn during the engagement. For brief engagements, the Markov assumption would appear to be reasonable.

Inclusion of the Markov assumption results in what may be described as a first-order model--the first approximation to ACM dynamics. The logical second-order model would consist of a conditioning of the state transition probabilities on the current state and the immediately prior state. Although the second-order model would provide a more accurate representation of ACM, it is unclear at this time that the added computational complexity is justified. Since research in this direction is incomplete, the discussion in this study will be limited to the evaluation of the first-order model.

The fourth assumption prevents certain mathematical pathologies from occurring and does not seem unreasonable on physical grounds. The fifth assumption rules out the possibility of a combatant winning and losing at the same time.

Methodology

Figure 1 is a flow diagram of the engagement model, assuming a start in state N. The evaluation fighter remains in state N for random time t_N, then transfers to either state O or state D according to the transition probabilities p(N, O) and p(N, D). Assuming transfer to O, the evaluation fighter remains in O for random time t_O, then transfers to state OW or state N in accordance with the probabilities p(O, OW) and p(O, N). Assuming transfer to OW, the evaluation fighter fires a weapon after random time t_{OW}. If the weapon is effective, the evaluation fighter wins, i.e., it transfers to state W, and the engagement ends. If the weapon is not effective, the transfer is back to state O, and the engagement continues similarly until either (1) one combatant wins, (2) a combatant runs out of weapons and breaks off (assumed to be possible), or (3) a time limit on engagement duration is exceeded, i.e., a combatant gets low on fuel and breaks off (assumed to be possible).

This collection of states (W, OW, O, N, D, DW, L) and associated transition probabilities are the ingredients for a Markov chain. States W and L are absorbing (the process stops when either of these states is entered) and the remaining states

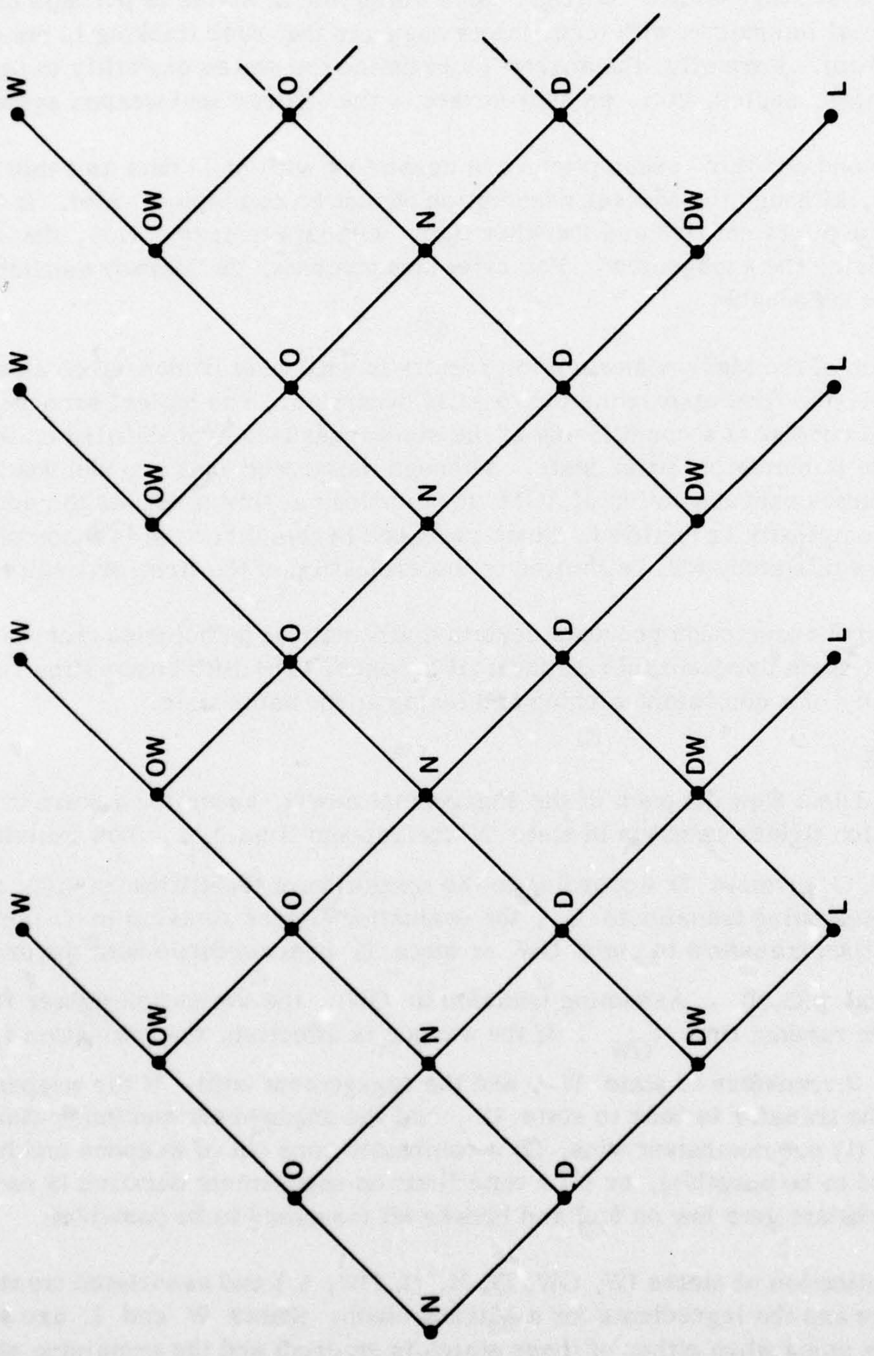

FIG. 1: STATE FLOW DIAGRAM

are transient (the probability that these states are occupied goes to zero as the number of transitions increases). When the time variables t_{OW} , t_{O} , etc., with associated densities f_{OW} , f_{O} , etc., are added, the result is a semi-Markov process (reference 7). The Markov chain structure allows us to calculate probabilities of absorption into W and L , but does not provide information about how long it takes to reach these states. The semi-Markov structure allows us to calculate the probabilities of absorption as a function of time, and also allows the analysis of engagements of fixed time duration T . This methodology is developed in appendix A.

A draw may occur in three ways: (1) the engaging fighter runs out of weapons, (2) the opponent runs out of weapons, or (3) the time limit T on engagement duration is exceeded, e.g., fuel constraints may require breakoff. It is assumed that either opponent may break off the engagement for one of these reasons, but does not otherwise do so. Using the original semi-Markov process as a building block, a new semi-Markov process can be constructed in which the events of a draw due to munition expenditure are defined as additional absorbing states. This development is also carried out in appendix A.

Although the emphasis in this report is on the analytical approach, it should be pointed out that this semi-Markov model lends itself to easy Monte Carlo simulation in that programming requirements are minimal and storage requirements are small. Such simulation allows easy estimation of the tactical MOEs mentioned previously.

Some of the common tactical MOEs used in ACM analysis are listed below:

- First firing probability: probability of getting the first shot.
- Engagement domination index: expected fraction of time in offensive or higher states (table 2).
- Engagement survivability index: expected fraction of time in neutral or higher states.
- Exchange ratio: ratio of the probability of win to the probability of loss.

All of these may be estimated by use of the Maneuver Conversion Model, and the first firing probability and exchange ratio by the Firing Sequence Model. However, the models are capable of providing an understanding of the ACM capability of a given aircraft that may be difficult to compress into a few statistics.

Results

To avoid security classification of this volume, fictitious data is used to illustrate the methodology. Volume II contains an analysis of ACM data gathered by VX-4 during the prosecution of CNO project Battle Cry (reference 6).

The time-in-state variables t_O, t_N, t_D are assumed to have lognormal density functions:

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma t} \exp\left\{ -\tfrac{1}{2} \left[\ln(t) - m \right]^2 / \sigma^2 \right\}$$

for $0 < t < \infty$ (reference 8). The parameters m and σ chosen were:

	f_O	f_N	f_D
m:	1.743	2.738	2.171
σ:	0.710	0.729	1.088

Lognormal distributions are commonly used to model time-in-state variables in many contexts (reference 9). Although there is no obvious physical reason why it should be so, lognormal distributions were reported to provide good fits to range data obtained during Battle Cry (reference 6).

The time variables for states OW and DW were modeled as having uniform densities on the time interval $[0, 30]$ (sec). This choice was based on informal interviews with pilots since data of actual firing delays was limited.

The above parameters result in means and standard deviations for the time variables as indicated below:

Variable	Mean (sec)	Standard deviation (sec)
t_{OW}	15.0	8.7
t_O	7.3	5.9
t_N	20.2	16.9
t_D	15.9	23.9
t_{DW}	15.0	8.7

The following state transition probabilities are used:

p (O, OW)	= .2	p (N, D)	= .2
p (O, N)	= .8	p (D, N)	= .75
p (N, O)	= .8	p (D, DW)	= .25

The probability of achieving a kill, given that a weapon was fired, is taken as 0.4 for both fighters, so that:

$$p\,(OW, W) = p\,(DW, L) = .4$$
$$p\,(OW, O) = p\,(DW, D) = .6$$

Results of the analysis for various combinations of missiles carried aboard and for various durations of engagement are summarized in table 3. The results are in the form of probability of win and loss, and probabilities of each of the three types of draw. Thus, the lower right-hand block gives these probabilities in the case of infinite engagement time duration and unlimited munitions as: probability of win equal to .76, probability of loss equal to .24, and probability of all three types of draw equal to 0.

As the engagement duration increases, the probability of win increases from .08 to its limiting value of .76. Figure 2 graphs this rise in probability of win as a function of time in more detail. Also shown are the probabilities of loss and draw as functions of time for this case.

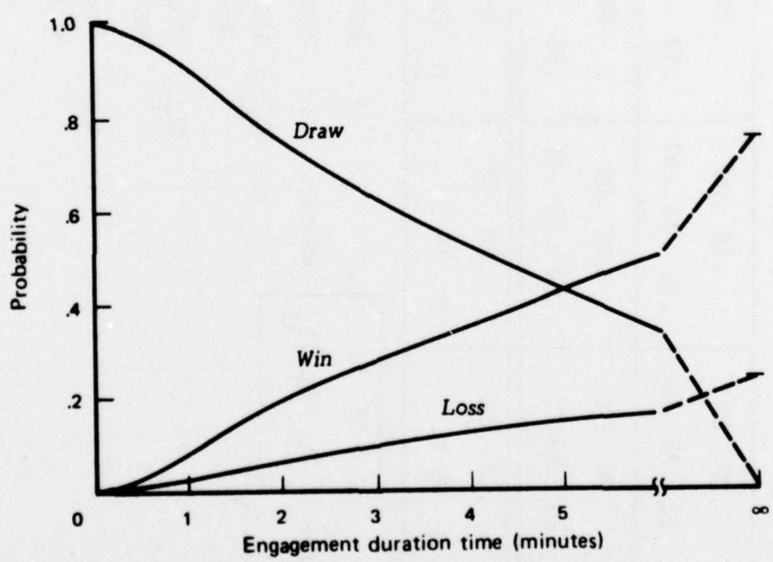

FIG. 2: PROBABILITIES OF WIN, LOSS, AND DRAW vs. ENGAGEMENT DURATION TIME

TABLE 3

ENGAGEMENT DURATION
(Minutes)

Number of missiles (each)	1	2	3	5	∞
1	.08 .02 .12 .75 .03	.16 .05 .25 .47 .07	.22 .07 .32 .29 .10	.27 .08 .40 .13 .12	.30 .10 .46 0 .14
2	.08 .02 .01 .89 0	.19 .06 .04 .70 .01	.28 .09 .08 .53 .02	.39 .12 .15 .31 .03	.51 .17 .26 0 .06
∞	.08 .02 0 .90 0	.20 .06 0 .74 0	.29 .09 0 .62 0	.43 .14 0 .43 0	.76 .24 0 0 0

Block entries are: $\boxed{\begin{matrix} W & L & \\ D_E & D_T & D_O \end{matrix}}$ where

W: Probability of win by evaluation fighter

L: Probability of loss by evaluation fighter

D_E: Probability of draw due to missile exhaustion of evaluation fighter

D_T: Probability of draw due to exceeding engagement duration

D_O: Probability of draw due to missile exhaustion of opposing fighter

Figure 3 displays the MOE "First firing probability" as a function of engagement duration time. As would be expected, the value p(O, N) = .8 gives the evaluation fighter a distinct advantage as far as this MOE is concerned. The methodology for the computation is given in appendix A.

The exchange ratio "Probability (win)/probability (loss)" is approximately 3 in the case of infinite time, regardless of initial missile loadout (assuming equal loadout). However, table 3 indicates that the exchange ratio is not constant with time.

Figure 4 displays the exchange ratio as a function of time for the case where each fighter has unlimited munitions. As the figure indicates, the exchange ratio rises rapidly to a peak around 25 seconds, then slowly levels off to its limiting value (3.09). This general behavior is typical for this set of engagement parameters, providing each fighter starts with the same number of missiles. This rapid rise in exchange ratio may be anticipated from figure 1 in which it is seen that the evaluation fighter has a probability of .16 of transitioning directly from N to O to OW, with associated mean time of 27.5 seconds (from the table of means and standard deviations). The corresponding probability of transition from N to D to DW is only .05, with associated mean time of 36.1 seconds.

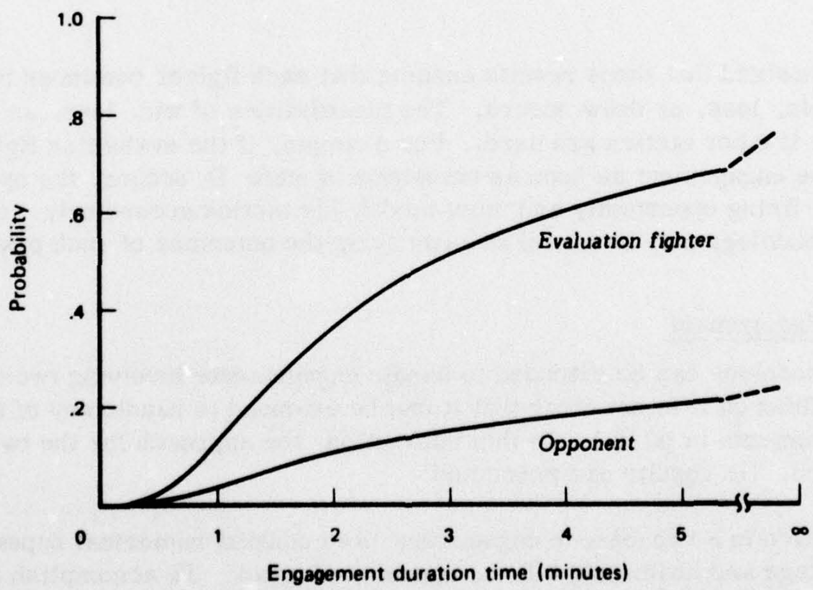

FIG. 3: PROBABILITY OF ACHIEVING THE FIRST FIRING vs. ENGAGEMENT DURATION TIME

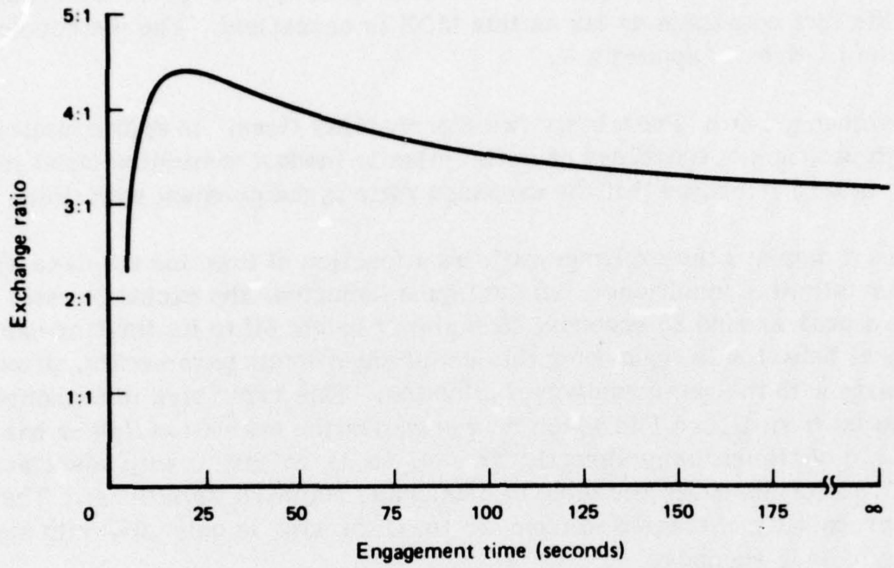

FIG. 4: EXCHANGE RATIO vs. ENGAGEMENT TIME

It is emphasized that these results assume that each fighter continues the engagement until a win, loss, or draw occurs. The probabilities of win, loss, and draw shift radically if other tactics are used. For example, if the evaluation fighter is free to break off the engagement as soon as transition to state D occurs, the opponent never gets a weapon-firing opportunity and must modify his tactics accordingly. Clearly, this modeling methodology may be useful in quantifying the outcomes of such proposed tactical concepts.

Two-on-One Engagement

This methodology can be extended to handle engagements involving two-on-one and one-on-two, although it is not clear that it can be extended to handle any of the other types of engagements in table 1. In this subsection, the approach for the two-on-one case is outlined. No results are presented.

The objective in a two-on-one engagement is to convert numerical superiority to a tactical advantage and ultimately kill the opposing aircraft. To accomplish this, the section fighters coordinate maneuvering to prevent an unfavorable one-on-one engagement. The simplest form of such a coordinated attack consists of one fighter actively engaging the opponent while the other free fighter monitors the engagement. In this way, the

free fighter may enter the engagement at an opportune moment. As an aircrew's experience in coordination increases, both section fighters will often maneuver simultaneously against the opponent in a coordinated attack. Such engagements may be analyzed by treating the fighter section as a tactical unit and applying the methodology for the one-on-one evaluation.

To use the one-on-one methodology, states are defined for the section as a whole. Table 4 provides criteria that might be applied to a two-on-one engagement to define the section states.

TABLE 4

ENGAGEMENT STATES FOR A TWO-ON-ONE ENGAGEMENT
(Relative to an evaluation section)

1. The section is in state OW when at least one member is in state OW and the other is higher than state DW.
2. The section is in state O when at least one member is in state O and the other is higher than state DW.
3. The section is in state N when both members are in state N.
4. The section is in state D when at least one member is in state D and the other is in state N or D.
5. The section is in state DW when at least one member is in state DW and the other is in less than state OW.
6. The section is in a tradeoff state when one member is in state OW and the other is in state DW.

Higher-Order Engagements

From a mathematical point of view, it would seem natural to continue the expansion of the number of states and apply the basic methodology to two-on-two and eventually more complex ACM engagements. Unfortunately, such an attack is inappropriate as the following comments indicate. First, the Maneuver Conversion Model emphasizes dynamic maneuvering and equates good maneuvering with success. In the simplest high-order (two-on-two) engagement, maneuvering performance is valued less in determining success than the use of sound tactics. Second, as the number of states increases, the model begins to lose its intuitive meaning to the aircrews. Since a primary goal of the modeling procedure is to assist aircrews assess ACM performance, the continued increase in model complexity is self-defeating. Alternative analytic models for engagements involving multiple aircraft have been developed and will be published separately.

FIRING SEQUENCE MODEL

The Firing Sequence Model is designed primarily for the evaluation of ACM engagements in which both combatants are armed with all-aspect weapons. The model quantifies the observed samples of simulated firing incidents in terms of probabilities of win, loss, and draw. Ideally, this model and the Maneuver Conversion Model should both be used to obtain an overall evaluation of the relative maneuvering performance of an aircraft as well as to estimate the contribution of all-aspect weapons to system effectiveness.

The Firing Sequence Model is an extension of usual methods of analyzing test-range engagement data. Typically, in the past, each engagement in a data set of n test-range engagements is examined to determine the probabilities $p_K(e)$, $p_L(e)$, $p_D(e)$ of win, loss, or draw, respectively, for each engagement, e, in the data set. The numerical averages:

$$(1/n \sum_j p_K(e_j) \,, \qquad (1/n) \sum_j p_L(e_j) \,, \qquad (1/n) \sum_j p_D(e_j)$$

are commonly taken as the estimates of the probabilities of win, loss, and draw. For large sample sizes, these averages could be expected to provide reasonably accurate estimates of the probability of a win, loss, or draw had real weapons been used. However, the time and expense required to obtain large samples on a test range does not appear to be justified or practical.

The Firing Sequence Model is designed to evaluate the interdependence of the observed firings to predict the relative frequency of occurrence of unobserved firing sequences. These sequences are then included in the computation of the probability of a win, loss, or draw according to their predicted weight or relative frequency.

Assumptions

Specific assumptions used in constructing the Firing Sequence Model are:

- Each test range datum, e, is a simulated firing sequence: $e = (s_1, s_2, \ldots, s_k)$. Each s_i is a zero, one, or two corresponding to a simulated missile firing by, respectively, the opposing fighter, the evaluation fighter, or both.
- For each engagement, e, the time of the start of the engagement and the times of the simulated firings are known.
- The likelihood that a combatant fires depends on which combatant fired in the previous incident, but is independent of all earlier firing incidents. That is, who shoots next depends only on who is shooting now, and not on prior firings.

The first two assumptions are not mathematical in nature, but merely stipulate the form of input data required for the model. It should be noted that whereas the Maneuver Conversion Model required a knowledge of engagement states between firings, the Firing Sequence Model does not. The second assumption is necessary if a complete analysis is desired, but it will be seen that some results are possible if the time data is incomplete. The third assumption is a Markov assumption made (1) for statistical convenience, and (2) because pilot interviews and preliminary examinations of range data suggest that it is approximately correct. The assumption cannot be strictly valid since it requires that the pilots do not tire and that missiles which miss do not inflict any damage. As with the Maneuver Conversion Model, the Markov assumption in the Firing Sequence Model may be considered as a first-order approximation of ACM dynamics.

Methodology

It may happen that in a given body of range data, the encounter $e_1 = (1, 0)$ occurs 20 times while the encounter $e_2 = (0, 1)$ occurs only twice. In such a case, it is clearly improper to average $p_K(e_1)$ with $p_K(e_2)$ using equal weights; they should be weighted with their respective frequencies of occurrence. Motivated by this observation, for each possible $e = (s_1, \ldots, s_k)$ let $f(e)$ be the probability that the engagement generates that precise sequence. If this function $f(.)$ were known, then the probability of a win by a combatant would be given by:

$$\sum_e p_W(e)\ f(e) \quad .$$

A direct estimation of $f(.)$ from range data is out of the question because of the requirements for the size of the sample. Appendix B, however, shows that estimation of $f(.)$ can be done with reasonable sample sizes if assumption three, the Markov assumption, can be invoked.

In brief, let $h(t)$ be the probability that an engagement will last t seconds if no firing incidents produce a kill. This function may be estimated from range data, or may be hypothesized in specific scenarios from an evaluation of the planned engagement tactics. Let $f(e\,|t)$ denote the conditional probability of observing the sequence $e = (s_1, \ldots, s_k)$, given that the engagement lasts t seconds. It follows that:

$$f(e) = \int_0^{\infty} f(e\,|t)\ h(t)\ dt$$

and hence that estimation of $f(.)$ reduces to estimation of $f(\,.\,|\,t)$.

Given the Markov assumption, and for $i, j = 0, 1, 2$, let $p(i \mid j)$ denote the probability of a firing of type i given that the previous firing was of type j. Let $p(i)$ be the probability that the first firing is of type i. Appendix B shows that:

$$f(e \mid t) = g(e \mid t)\, p(s_1) \prod_{j=2}^{\ell(e)} p(s_j \mid s_{j-1})$$

where $g(e \mid t)$ is a rather complicated expression involving the convolution of distribution functions and $\ell(e)$ is the length of the sequence. The function $g(e \mid t)$ may be interpreted as the probability that a sequence has length $\ell(e)$, given that the first $\ell(e)$ terms are $s_1, \ldots, s_{\ell(e)}$ and given the engagement time of t seconds. This function will be referred to as the sequence length frequency function. Multiplication of this double conditional probability by the probability of observing the entries $s_1, \ldots, s_{\ell(e)}$ yields the firing sequence frequency function.

Results

To illustrate the methodology, the firing sequences generated in a simulation of 200 engagements are analyzed. A thorough analysis of the data is not presented; rather, a partial analysis is given that illustrates the technique and highlights the need for additional research in certain areas.

The data was generated by using a Monte Carlo simulation of the Maneuver Conversion Model, and is presented in appendix B. The engagements were automatically terminated after 3 minutes, with combatants given four air-to-air missiles each. Other parameters were the same as those used to produce table 3. Time data was not recorded for these 200 firings, so that the full generality of the methodology is not illustrated.

With time data absent, the estimation of $g(e \mid t)$ requires additional assumptions. If the distributions of the times between firings are hypothesized or otherwise known, the expression for $g(e \mid t)$ can be evaluated for all possible sequence (maximum length eight) in principle, and the win probability of each sequence can be estimated. This would be a lengthy computational procedure unless some of the work can be done analytically. The work is greatly reduced if it can be assumed that the times between firings are identically distributed, given s_1. In this case, $g(e \mid t)$ becomes $g(\ell(e) \; s_1, t)$ and this latter conditional probability can be estimated directly from the firing sequence data by computing the frequency of occurrence of sequences of length 0, 1, 2, etc., as a function of s_1 and time. The work is further reduced if there is reason to believe that $g(e \mid t)$ is independent of s_1.

Table 6 shows the formulas derived in the text and appendix B, specialized for this data. No simulataneous firings occurred, so that firings of type 2 are absent. $g(e \mid t)$ is estimated under the assumption that the times to first firing and the times between firings are all identically distributed. This is not a good assumption for this data, but it is not clear that the sample size is large enough to support a more realistic assumption. Further research needs to be done on sample size requirements versus assumptions.

Table 7 displays the estimates of the parameters listed in table 6. The estimators are binomial or multinomial random variables and confidence limits on the precision of estimation can be gotten from table 7 if desired. In principle, these probabilities could be calculated exactly, since the statistics of the underlying Maneuver Conversion Model are known. This would be an extremely lengthy procedure and was not done since confidence limits are available.

Table 8 displays the resulting frequency function $f(.)$. Since all the engagements were ended at 3 minutes (no draws), the integration:

$$f(e) = \int_0^\infty f(e \mid t)\, h(t)\, dt$$

is vacuous and these numerical results are all conditioned on an engagement length of 3 minutes. For example, in table 8 $f(e)$ for $e = (0,0,0)$ is given by:

$$g(3 \mid 3 \text{ min})\, p(0)\, p(0 \mid 0)\, p(0 \mid 0) = (.06)\,(.282)\,(.6)^2 = .0061 \quad .$$

Note that if $f(e)$ were estimated only by frequency of occurrence of e , the sequences (0, 0, 0), (1, 0, 1), (0, 1, 0), and (0, 1, 1) would have the same $f(e)$, whereas by this method the values of $f(e)$ are all distinct, and properly so.

Table 9 displays probabilities of win, loss, draw for all sequences through length four, assuming a missile kill probability of 0.4, given a firing. The probabilities are all referenced to the evaluation fighter and are calculated by adding up all the ways the fighter can win or lose.

The final results are probabilities of win, loss, and draw of .26, .11, and .63, respectively. A direct comparison with the results of table 3 shows that these values are reasonable. In this particular case, the results are identical (slight difference in the third digit) with those obtained when $f(e)$ is estimated directly by frequency of occurrence of e . It is not hard to conceive of setups in which the two methods of estimation will produce dissimilar results; further research is therefore needed to enable the analyst to tell when estimation by frequency of occurrence is adequate and when it is not.

TABLE 6

ESTIMATION OF FIRING SEQUENCE MODEL STATISTICS[1]

1. Sequence length frequency function. $g(.|.)$

   $g(\ell|3)$ = number of sample sequences of length ℓ divided by the sample size, for $\ell = 1, \ldots, 4$.

2. First firing probabilities. For $i = 0, 1$

   $p(i)$ = number of sample sequences for which the first entry is i , divided by the number of sample sequences with at least one firing or opportunity.

3. Conditional firing probabilities. For $i, j = 0, 1$

   $p(i|j)$ = number of times an entry j is followed by the entry i in the sample, divided by the number of times an entry j was followed by either 0 or 1.

Notation:

1 - Firing by the evaluation fighter.
2 - Firing by the opposition fighter.

[1] Formulas specialized for dealing with the simulated data. That is: (1) all engagements lasted 3 minutes, so that only $g(.|3)$ need be calculated; (2) the maximum observed sequence length is 4.

TABLE 7

SAMPLE COMPUTATION OF FIRING SEQUENCE MODEL STATISTICS

1. Sequence length frequency function. g(. |3)

Sequence length ( )	g(. | 3)
0	58/200 = .290
1	79/200 = .395
2	48/200 = .240
3	12/200 = .060
4	3/200 = .015
5	0/200 = 0

2. First firing probabilities

P (Evaluation fighter attains first firing) = 102/142 = .718

P (Opposition fighter attains first firing) = 40/142 = .282

3. Conditional firing probabilities.

Conditioned event[a]	Probability
0|0	p (0 | 0) = 15/25 = .60
1|0	p (1 | 0) = 10/25 = .40
0|1	p (0 | 1) = 13/56 = .23
1|1	p (1 | 1) = 43/56 = .77

Notation:

i|j - Firing of type i given that the previous firing was of type j for i, j = 0, 1

0 - Opposition fighter

1 - Evaluation fighter

[a] Computations based on the formulas in table 6 and the simulated data listed in table B-1 of appendix B.

TABLE 8

ESTIMATED FREQUENCY FUNCTION

e	Number of occurrences	f(e)	e	Number of occurrences	f(e)
∅	58	.2900	(1,1,1,1)	2	.0049
(0)	21	.1114	(1,1,1,0)	0	.0015
(1)	58	.2836	(1,1,0,1)	1	.0008
(1,1)	28	.1327	(1,1,0,0)	0	.0011
(1,0)	4	.0396	(1,0,1,0)	0	.0002
(0,1)	6	.0271	(1,0,0,1)	0	.0006
(0,0)	10	.0406	(1,0,1,1)	0	.0008
(1,1,1)	2	.0255	(1,0,0,0)	0	.0009
(1,0,0)	3	.0059	(0,1,1,1)	0	.0010
(1,1,0)	3	.0076	(0,1,1,0)	0	.0003
(1,0,1)	1	.0040	(0,1,0,1)	0	.0002
(0,1,0)	1	.0016	(0,1,0,0)	0	.0002
(0,1,1)	1	.0052	(0,0,1,0)	0	.0002
(0,0,1)	0	.0041	(0,0,0,1)	0	.0006
(0,0,0)	1	.0061	(0,0,1,1)	0	.0008
			(0,0,0,0)	0	.0009

TABLE 9

FIRING SEQUENCE PROBABILITIES OF
WIN, LOSS, AND DRAW
(Missile kill probability = .4)

Sequence	Probability of Win	Loss	Draw	Sequence	Probability of Win	Loss	Draw
				<1,1,1,1>	.870	0	.130
∅	0	0	1.0	<1,1,1,0>	.784	.086	.130
<0>	0	.4	.6	<1,1,0,1>	.726	.144	.130
<1>	.4	0	.6	<1,1,0,0>	.640	.230	.130
<0,0>	0	.640	.360	<1,0,1,0>	.544	.326	.130
<1,1>	.640	0	.360	<1,0,0,1>	.486	.384	.130
<0,1>	.240	.400	.360	<1,0,1,1>	.630	.240	.130
<1,0>	.400	.240	.360	<1,0,0,0>	.400	.470	.130
<1,1,1>	.784	0	.216	<0,1,1,1>	.470	.400	.130
<1,0,0>	.400	.384	.216	<0,1,1,0>	.384	.486	.130
<1,1,0>	.640	.144	.216	<0,1,0,1>	.326	.544	.130
<1,0,1>	.544	.240	.216	<0,1,0,0>	.240	.630	.130
<0,1,0>	.240	.544	.216	<0,0,1,0>	.144	.726	.130
<0,1,1>	.384	.400	.216	<0,0,0,1>	.086	.784	.130
<0,0,1>	.144	.640	.216	<0,0,1,1>	.230	.640	.130
<0,0,0>	0	.784	.216	<0,0,0,0>	0	.870	.130

DATA COLLECTION

Data for both models is collected by use of an instrumented air combat range. Engagements are simulated and the various sensors at the range monitor the position and altitude of each aircraft so that an accurate continuous-time record of the engagements can be reconstructed. A group of evaluation aircraft and pilots is designated and matched against a corresponding group of opposition aircraft and pilots. Ideally, the matching is done so that each pilot in each group is matched against each of the opposing pilots the same number of times. Usually, the differences in individual aircraft within a group are not such that individual pilot/aircraft combinations need be randomized. In practice, schedules are not always met, certain matchings will not always occur, and for various such reasons, the resulting body of data may be incomplete or defective. In such cases, it may be possible to extract desired estimates by using experimental design techniques. This area is not explored in this study.

It is assumed that the testing is done with some specific goal in mind, i.e., to measure the ACM performance of the specific evaluation aircraft against the specific opposition aircraft under certain conditions and involving specific types of tactics. These conditions and tactics will usually specify the initial conditions for the engagements, or at least the probability distribution for the initial conditions.

MANEUVER CONVERSION MODEL

In order that model parameters be estimated from the engagement data, precise definitions of each state must be given in terms of relative geometry, speed, etc. The details of the definitions will vary with aircraft types, and should be laid out to the mutual satisfaction of the pilots and analysts prior to the testing. The details of this procedure will not be explored here; examples may be found in reference 6. Given such definitions, the analyst can determine changes of state from the reconstructed engagements. Ideally, this process is entirely automated. Simulated firing incidents, along with engagement start, duration, etc., may be identified from pilot voice tapes.

Figure 5 and table 10 summarize the data to be extracted from the reconstruction of each engagement and the calculations made from the data. Ideally, these calculations are entirely automated, although the forms have actually been used for manual reconstruction and analysis.

It is extremely difficult to predict ahead of time how much data is "enough." It may be possible to estimate how many state transitions are expected in an "average" encounter, with each transition providing a data point for estimating transition probability as well as time-in-state density. In that case, it would be possible to estimate how many encounters are needed. After the fact, it is straightforward to calculate what estimation accuracy has been obtained for each of the input parameters. It is less straightforward, but possible (e.g., reference 10), to determine the effect of the imprecise estimation on the final numerical results. This is a topic which needs further research, but is not explored here.

DATE: ______________________ OP. NO./RUN ______________________

I. *DEFINITION OF COMBATANT CLASSES*

	FIGHTER 1	FIGHTER 2
AIRCRAFT TYPE		
WEAPON LOAD		
CREW		
MISSION CODE		
ENGAGEMENT TACTICS RESTRICTIONS		

II. *TACTICAL ENGAGEMENT SUMMARY*

INITIAL SETUP (Including Altitude/Airspeed) ______________________

TIME	ENGAGEMENT STATE	REMARKS AND FIRING INDICATIONS

III. *ENGAGEMENT EVALUATION*

A. STATE CONVERSION SUMMARY

CONVERSION OPTION	NUMBER OF CONVERSIONS	CONVERSION OPTION	NUMBER OF CONVERSIONS
OFF WPN TO OFF		OFF TO OFF WPN	
OFF TO NEUTRAL		NEUTRAL TO OFF	
NEUTRAL TO DEF		DEF TO NEUTRAL	
DEF TO DEF WPN		DEF WPN TO DEF	

B. TIME IN ENGAGEMENT STATE

STATE	NUMBER OF TIMES IN STATE	DURATIONS *(List sequentially)*	TOTAL TIME
OFFENSIVE WEAPON			
OFFENSIVE			
NEUTRAL			
DEFENSIVE			
DEFENSIVE WEAPON			

C. WEAPON EXPENDITURE SUMMARY

	FIGHTER 1 WEAPON TYPE 1	2	3	4	FIGHTER 2 WEAPON TYPE 1	2	3	4
OPPORTUNITIES								
VALID FIRINGS								
INVALID (OUT-OF-ENVELOPE) FIRINGS								

FIG. 5: ONE-ON-ONE ENGAGEMENT RECONSTRUCTION AND EVALUATION FORM

DATE: Enter calendar date of operation.

OP NO./RUN: Enter operation number and engagement number.

A/C TYPE: Enter aircraft type and model.

WEAPON LOAD: Enter number and type of each on-board air-to-air weapon.

CREW: Enter crew names or serial numbers.

MISSION CODE: Enter 1 for aggressive air combat, and 2 for defensive, survival maneuvering.

INITIAL TACTICAL SETUP: Enter the relative tactical position, altitude, and airspeed of the engaging aircraft.

TIME/STATE SUMMARY: Starting at time t = 0, enter the initial engagement state. At each state conversion, enter the time of conversion and the new engagement state. Include firing indications in the "remark" column with appropriate time indication.

STATE CONVERSION SUMMARY: For each conversion option (neutral to defensive, neutral to offensive, etc.), enter the number of observed engagement conversions.

TIME IN ENGAGEMENT STATES: For each engagement state, enter the number of times in that state and list the times in state sequentially in the "duration" column.

WEAPON EXPENDITURE SUMMARY: For each fighter and each weapon type, enter the number of weapon opportunities, the number of valid (in-envelope) firings, and the number of invalid (out-of-envelope) firings.

FIGURE 5 (Continued)

TABLE 10

ESTIMATORS FOR MODEL PARAMETERS AND MOEs

$P(X, Y) = \dfrac{\text{Number of conversions from (X) status to (Y) status}}{\text{Number of times in (X) status}}$

F(X, ·) = Empirical distribution describing time in state X

p_{FF} = Ration of number of engagements in which evaluation fighter achieves first firing to number of engagements

EDI = Fraction of total engagement time spent in state O or DW

ESI = Fraction of total engagement time spent in state N or higher

FCI = Fraction of engagements in which evaluation fighter is first combatant to obtain a weapon opportunity

Legend:

- X, Y - Defensive weapon, defensive, neutral, offensive, offensive weapon
- p_{FF} - Probability of first firing in engagement
- EDI - Engagement domination index
- ESI - Engagement survivability index
- FCI - Fighter capability index

FIRING SEQUENCE MODEL

The data gathered for the Maneuver Conversion Model is more than sufficient for estimating Firing Sequence Model parameters. The only data required for estimating these parameters are the times for each simulated firing and the type of firing (whether done by the evaluation aircraft, opposition aircraft, or both simultaneously).

The comments in the preceding subsection about being unable to predict how much data is "enough" apply here also. Here, fewer parameters are being estimated than for the Maneuver Conversion Model; on the other hand, not every encounter will result in a data point (no simulated firing) for the Firing Sequence Model.

AREAS OF APPLICATION

These models have potential application in several areas. These include analyzing fighter system or weapon system flyoffs, evaluating aircrew ACM proficiency, and quantifying the extent to which our aircraft simulates an enemy threat aircraft. These topics are discussed below.

In making the decision about buying one of two competing aircraft/systems, the ability to quantify their relative ACM performance may be a valuable input. The models may be used to evaluate a controlled flyoff against each other or against specific opposition aircraft under test-range conditions.

These models could be used by the Fighter Weapons School to grade the proficiency and improvement of students as they progress through training. If certain students have ACM weaknesses, this will show up in the model outputs. and careful examination of the model parameters for their engagements may help pinpoint precise causes.

The unavailability of actual threat aircraft has hampered the training of fleet aircrews for ACM encounters with these threat aircraft. As a result, fleet aircrews must practice against available aircraft, in the hope that such aircraft provide an adequate simulation of the threat aircraft. These models provide a way of quantifying how well candidate aircraft simulate a specific threat aircraft in ACM. After a candidate aircraft is flown against the threat aircraft on the test range, the resulting estimated model parameters quantify the relative abilities of the aircraft.

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APPENDIX A

SEMI-MARKOV ANALYSIS METHODOLOGY
FOR THE MANEUVER CONVERSION MODEL

APPENDIX A

SEMI-MARKOV ANALYSIS METHODOLOGY FOR THE MANEUVER CONVERSION MODEL

INTRODUCTION

It is desired to compute the probabilities of a win (transfer to W), a loss (transfer to L), or of a draw, each as a function of time in a given time interval [O, T]. Three kinds of a draw are permitted: (1) the evaluation fighter runs out of missiles and breaks away, (2) the opposing fighter runs out of missiles and breaks away, and (3) the time limit T on length of engagement is exceeded, e.g., fuel constraints require breakoff. In practice, a combatant may have some difficulty in safely breaking off the engagement at will, but this analysis assumes that it is possible. As described in the text, states W and L are absorbing states; hence, the probabilities of win and loss are probabilities of absorption. It will be shown that the probabilities of occurrence of the first two types of draws can also be defined and calculated as absorption probabilities.

As discussed in the text, states W, OW, O, N, D, DW, and L, with associated transition probabilities, define a Markov chain. The time-in-state variables (or simply "time variables") and associated densities provide the additional ingredients to define a semi-Markov process. The methodology derived here can be used to handle the analysis of this semi-Markov process. This process, however, has five transient states (OW, O, N, D, DW) which, in this particular case, is unnecessarily large. We will show that by restricting attention to states W, O, D, and L, we can still talk about probability of absorption into states W and L, but have only two transient states. Such a reduction in the number of states is advantageous computationally, but is not always possible.

Next, the general methodology is developed as two cases: Case I provides the basic methodology for a semi-Markov process with an arbitrary (but finite) number of states under the assumption of unlimited munitions by both combatants. This assumption is not realistic for analysis of ACM; however, the results are a stepping stone toward the solution of the more realistic case II with limited munitions. Further, the results of case I are of interest in themselves for the analysis of combat situations in which munitions are essentially unlimited, e.g., a wrestling match.

Finally, methodology is derived for calculating some of the MOEs discussed in the main body of the test.

REDUCTION OF DIMENSION

Here, it is shown that we need consider only the states W, O, D, and L for the purposes of our analysis. This reduction in dimension from seven to four is made for computational convenience, and the reader who is interested only in the general methodology may wish to pass over this subsection.

We will speak of the original process with seven states as the "old" process. We pass to a new semi-Markov process obtained by regarding two transitions of the old process as a single transition. The matrix of transition probabilities for the new process is obtained by squaring the matrix for the old process. When this is done, we see that in the new process, the transient states split into the two disjoint sets, (O, D) and (OW, N, DW). This split occurs because of the requirement that only transitions to adjoining states are allowed. Thus, if the system starts in one of the transient states O or D in the new process, it will oscillate among these two states until absorption, i.e., the states OW, N, and DW will never be entered (in the new process). Hence, by passing to this new process, we can still talk about probability of absorption into W and L but have only two transient states. Providing that we can write down the necessary transfer probabilities and density functions, this embedded semi-Markov process will serve our purposes. We sacrifice our ability to obtain any results pertaining to the missing three transient states OW, N, and DW (e.g., first passage times), but in our analysis we are not concerned with these states.

Restricting our attention to the embedded semi-Markov process with states W, O, D, and L, we must write down the associated transition probabilities and density functions. These can be written down by inspection from figure 1 (in the main text):

$$
\begin{aligned}
p(W,O) &= p(W,D) = p(W,L) = 0\\
p(L,W) &= p(L,O) = p(L,D) = 0\\
p(W,W) &= p(L,L) = 1 \quad .
\end{aligned}
$$

$p(O,W)$ equals $p(O,OW)$ multiplied by the probability of kill given a missile firing; $p(O,O)$ equals $p(O,OW)p(OW,O) + p(O,N)p(N,O)$, etc. Similarly, the corresponding transition times between states have densities that can be expressed in terms of the density functions f_S, $S = OW, O, N, D, DW$. Thus, the time from state O to W equals $(t_O + t_{OW})$ and, hence, has density function $f_O * f_{OW}(\cdot)$, where $*$ denotes the convolution operation. The time variables for passage from O to D and from D to O have similar densities. The time for passage from O to O, denoted $t_{O,O}$, is treated differently.

$$
\begin{aligned}
\text{Prob}\,(O \to O; t_{O,O} = t) &= \text{Prob}(O \to OW \to O: t_O + t_{OW} = t)\\
&\quad + \text{Prob}(O \to N \to O; t_O + t_N = t) \quad .
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
p(O,O)f_{O,O}(\cdot) &= p(O,OW)p(OW,O)f_O * f_{OW}(\cdot)\\
&\quad + p(O,N)p(N,O)f_O * f_N(\cdot) \quad ,
\end{aligned}
$$

and $f_{D.D}(.)$ is similarly derived. We now have all that is required for the definition of a semi-Markov process with states W, O, D, and L . Figure A-1 displays the associated flow diagram and transition probabilities for the numerical example. Since N is not one of the states in this new process, we must think of this new process as starting in O with probability .8 and in D with probability .2 .

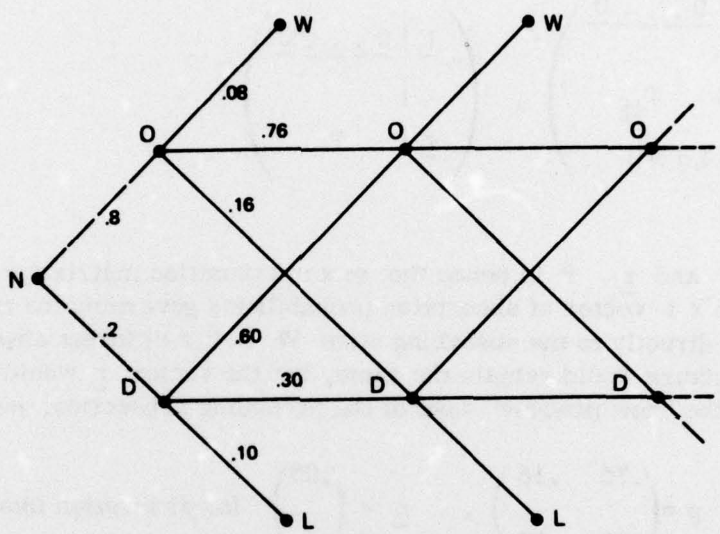

FIG. A-1: FLOW DIAGRAM AND TRANSITION PROBABILITIES FOR THE EMBEDDED SEMI-MARKOV PROCESS (NEW PROCESS)

CASE I: UNLIMITED MUNITIONS

In this case, we concern ourselves with the probability of a win by the evaluation fighter. The probability of a loss may be computed with this same methodology since it is equal to the probability of the opposition fighter achieving a win. Munitions are considered unlimited, so that the only kind of draw in this case results from exceeding the time limit on the engagement. Formally, we have a semi-Markov process with a single absorbing state W and two transient states O and D as described in the preceding subsection ("transient" means that absorption must occur eventually, i.e., the process cannot oscillate in states O and D indefinitely). We wish to compute the probability of absorption as a function of time. In the following text, the methodology is derived for the case of m transient states, since the general result will be needed in case II.

We commence with a Markov chain and extend to a semi-Markov process. Suppose we have a Markov chain with m transient states and some unspecified number of additional absorbing states. We select one of the absorbing states and calculate the probability of absorption. Let the absorbing state be denoted state W and let the transient states be 1, . . . , m with corresponding transition probabilities p_{ij} . The transition matrix for these states may be written as:

$$\left(\begin{array}{c|c} \underline{1} & \underline{0 \; . \; . \; . \; 0} \\ p_{1W} & \\ \cdot & p_{ij} \\ \cdot & \\ \cdot & i, j > 0 \\ p_{mW} & \end{array}\right) = \left(\begin{array}{c|c} \underline{1} & \underline{0 \; . \; . \; . \; 0} \\ & \\ \underline{r} & P \end{array}\right)$$

which defines P and $\underline{r}$. P is hence the m x m transition matrix for the transient states, and $\underline{r}$ is the m x 1 vector of absorption probabilities governing the transitions from the transient states directly to the absorbing state W . If a different absorbing state had been chosen, the structure would remain the same, but the vector $\underline{r}$ would be correspondingly different. For the "new process" case of the preceding subsection, we have:

$$m = 2, \quad p = \begin{pmatrix} .76 & .16 \\ .60 & .30 \end{pmatrix}, \quad \underline{r} = \begin{pmatrix} .08 \\ 0 \end{pmatrix} \text{ for absorption into W ,}$$

and

$$\underline{r} = \begin{pmatrix} 0 \\ .1 \end{pmatrix} \text{ for absorption into L .}$$

The formula for absorption in this setup is well known (reference 7) and is given by:

$$\underline{w} = \underline{r} + P\underline{w} \tag{A-1}$$

or

$$(I-P)\underline{w} = \underline{r} \tag{A-2}$$

where I is the m x m identity matrix and $\underline{w}$ is the m x 1 vector of components w_i , the probability of ultimate absorption given start in transient state i . Solving equation A-2, we get $\underline{w} = (I-P)^{-1}\underline{r}$, and the total probability of absorption is $\underline{s}'\underline{w}$, where the ' notation denotes matrix transposition. We obtain the result for any other absorption state by replacing $\underline{r}$ as appropriate. It is emphasized that this result is for the Markov chain only. The resulting absorption probability corresponds to infinite time and an infinite number of transitions.

We consider now the semi-Markov process. The transitions between states i, j take time t_{ij} , a random variable with density $f_{ij}(.)$. Let $p_{ij}(t) = p_{ij}f_{ij}(t)$ and let P(t) denote the matrix with entries $p_{ij}(t)$. It is convenient to let $\underline{s}(t)$ denote the initial state vector with components $s_i(t)$ to allow for the case that the process needs time to arrive at the initial state. For the new process model, $s_1(t) = 0.8f_N(t)$ and $s_2(t) = 0.2f_N(t)$. Similarly, let t_i denote the random time to final absorption from transient state i , and let $w_i(t) = w_i f_i(t)$, with f_i being the density of t_i . Let $\underline{w}(t)$ be the vector with components $w_i(t)$ and let $\underline{r}(t)$ be the vector with components $r_i(t) = P_{iW}(t)$. The formula for absorption into W as a function of time is well known (reference 11) and is given by:

$$\underline{w}(t) = r(t) + P(t)\underline{w}(t) \quad . \tag{A-3}$$

Solution of equation (A-3) requires numerical methods and was performed with the methodology in reference 11. Finally, the expression $\underline{s}'\underline{w}$ gives the probability of final absorption as a function of time.

The similarity in form of equations (A-2) and (A-3) is not accidental. We may, in effect, use the same notation for analysis of the Markov chain and for the semi-Markov process providing that the given definitions and conventions for the semi-Markov case are observed. This will be useful in case II -- we derive results for the Markov chain, then obtain the results for the semi-Markov process by reinterpretation of the notation.

CASE II: LIMITED MUNITIONS

The preceding case deals with the situation in which absorption from OW to W occurs, with no regard for how many passages from O to OW to O may occur before final absorption. In reality, each passage from OW to O corresponds to an expenditure of one missile (or salvo, or burst), and each aircraft contains a finite number of missiles. This subsection extends the preceding methodology to cover the limited munitions case.

We examine, first, the loadout of one missile for each combatant. In addition to the absorbing states W and L , we define absorbing states D_E and D_O corresponding to draws for the evaluation fighter and opponent, respectively, resulting from missile exhaustion. Specifically, we define the sequence O→ OW→ O to be absorbing into D_E corresponding to a missile firing by the evaluation fighter with negative results. Similarly, D→ DW→ D is absorbing into D_O . We see from figure 1 (main text) that the appropriate matrix of transition probabilities for the transient states is now:

$$P = \begin{pmatrix} .64 & .16 \\ .60 & .15 \end{pmatrix}$$

with absorption states and probabilities:

$$W: \begin{pmatrix} .08 \\ 0 \end{pmatrix}, \quad D_E: \begin{pmatrix} .12 \\ 0 \end{pmatrix}, \quad D_O: \begin{pmatrix} 0 \\ .15 \end{pmatrix}, \quad L: \begin{pmatrix} 0 \\ .1 \end{pmatrix} \quad .$$

Solution of equation (A-3) with these values gives the result for a dogfight of infinite time duration, each fighter having one missile.

We extend now to the situation that the evaluation fighter has two missiles while the opponent has only one missile. The approach is straightforward--we extend the number of states so that the number of remaining missiles can be distinguished. Thus, the evaluation fighter remains in states 1 and 2 (O and W) until he fires one missile. At this point, he either wins or the missile fails and a transition back to O occurs. At this point, states O and W are treated as states 3 and 4, respectively. Similarly, the system remains in states 3 and 4 until the evaluation fighter fires another missile, resulting either in a win or a transition back to state O , now regarded as absorbing (draw, D_E). Hence, the transition matrix of transition probabilities for the transient states is now:

$$\begin{pmatrix} .64 & .16 & .12 & 0 \\ .60 & .15 & 0 & 0 \\ 0 & 0 & .64 & .16 \\ 0 & 0 & .60 & .15 \end{pmatrix}$$

with absorption states and absorption probability vectors:

$$\underline{r}_W = \begin{pmatrix} .08 \\ 0 \\ .08 \\ 0 \end{pmatrix}, \quad \underline{r}_{D_E} = \begin{pmatrix} 0 \\ 0 \\ .12 \\ 0 \end{pmatrix}, \quad \underline{r}_{D_O} = \begin{pmatrix} 0 \\ .15 \\ 0 \\ .15 \end{pmatrix}, \quad \underline{r}_L = \begin{pmatrix} 0 \\ .1 \\ 0 \\ .1 \end{pmatrix} \quad .$$

In this situation, m = 4. Solution of equation (A-3) with these values gives the results for a dogfight of infinite duration, the evaluation fighter having two missiles, the opponent one.

The extension to more missiles on each of the fighters may, in principle, be carried out as above. The number of states increases rapidly, however, as the number of missiles is increased, and it is easy to make mistakes while working out the transition matrix. An

alternate approach that avoids this difficulty is presented below. We note that the above transition matrix can be written in the form $\begin{pmatrix} P & F10 \\ 0^* & P \end{pmatrix}$, where:

$$P = \begin{pmatrix} .64 & .16 \\ .60 & .15 \end{pmatrix}, \qquad F10 = \begin{pmatrix} .12 & 0 \\ 0 & 0 \end{pmatrix}, \qquad 0^* = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} .$$

With this notation, equation (A-3) becomes the matrix system:

$$M\begin{pmatrix} w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} .08 \\ 0 \end{pmatrix}$$

$$M\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} .08 \\ 0 \end{pmatrix} + F10\begin{pmatrix} w_3 \\ w_4 \end{pmatrix} \tag{A-4}$$

using r_W as the absorption probability vector, with $M = I - P$. This is a system of recursive equations that can be solved directly in the Markov chain situation or by using the methodology developed for the semi-Markov situation in case I. With the proper choice of notation and proper way of thinking, this matrix system becomes "obvious" and can be written down immediately.

Instead of thinking in terms of states 1, 2, 3, 4, etc., as above, it is more useful to regard the numbers of missiles on each fighter as state indicators. For example, let state (2/1) denote the condition that the evaluation fighter has two missiles remaining while the opposition fighter has only one. In the above example, there are only two such states, (2/1) and (1/1). While the system is in state (2/1) or (1/1), it transitions in its "sub-states" O and D according to the probability matrix P . The matrix F10 is the matrix of transition probabilities for transitioning from (2/1) to (1/1). (The name "F10" means a firing of one missile by the evaluation fighter, and zero missiles by the opposing fighter.) We let $\underline{w}_{2/1}$ denote the probability of ultimate absorption from state (2/1) and let $\underline{r}_{2/1}$ denote the probability of direct absorption from (2/1), with corresponding definitions for $\underline{w}_{1/1}$ and $\underline{r}_{1/1}$.

Following the reasoning behind equation (A-2), we think "$\underline{w}_{2/1}$ equals the probability of immediate absorption from (2/1), plus the probability of making one transition within (2/1) then being absorbed ultimately, plus the probability of transiting to (1/1) then being absorbed ultimately." Hence, we write immediately:

$$\underline{w}_{2/1} = \underline{r}_{2/1} + P\underline{w}_{2/1} + F10\underline{w}_{1/1} .$$

Regrouping:

$$M\underline{w}_{2/1} = \underline{r}_{2/1} + F10\underline{w}_{1/1} \quad . \qquad \text{(A-5)}$$

The only unknown on the right-hand side is $\underline{w}_{1/1}$. We reason "$\underline{w}_{1/1}$ equals the probability of immediate absorption from (1/1) plus the probability of a single transition within (1/1) followed by ultimate absorption from (1/1)." Hence:

$$\underline{w}_{1/1} = \underline{r}_{1/1} + P\underline{w}_{1/1}$$

or

$$M\underline{w}_{1/1} = \underline{r}_{1/1} \quad . \qquad \text{(A-6)}$$

We then see that equations (A-5) and (A-6) correspond to equation (A-4).

We let F01 be the matrix of transition probabilities corresponding to zero missiles fired by the evaluation fighter and one missile fired by the opposing fighter. The set of recursive matrix equations for the case that each fighter has two missiles becomes:

$$\begin{aligned} M\underline{w}_{2/2} &= \underline{r}_{2/2} + F10\underline{w}_{1/2} + F01\underline{w}_{2/1} \\ M\underline{w}_{1/2} &= \underline{r}_{1/2} + F01\underline{w}_{1/1} \\ M\underline{w}_{2/1} &= \underline{r}_{2/1} + F10\underline{w}_{1/1} \\ M\underline{w}_{1/1} &= \underline{r}_{1/1} \quad . \end{aligned} \qquad \text{(A-7)}$$

The general procedure for arbitrary missile loadings (m/n) is to write the matrix equation for $\underline{w}_{m/n}$, see what is unknown on the right-hand side after regrouping, continue writing one equation for each unknown, arriving finally at $M\underline{w}_{1/1} = \underline{r}_{1/1}$. The equations are solved recursively, starting at $\underline{w}_{1/1}$ and working back to $\underline{w}_{m/n}$. The final answer becomes $\underline{s}'\underline{w}_{m/n}$ in this notation, where $\underline{s}$ is the initial state probability vector.

CALCULATION OF CERTAIN MOEs

In the main text, various MOEs were listed and it was commented that the Maneuver Conversion Model could compute each of them. The additional methodology for doing those computations is given here.

The existing methodology is sufficient for calculating the first firing probability (probability of getting the first shot). In the case I model, the probability of kill, given a weapon firing, is set equal to unity, in which case the probability of a win is equal to the probability of getting the first shot.

The other MOEs are expressed as the expected fraction of time in various states. To calculate the expected amount of time in a given state for an engagement of duration T, we proceed as follows: As in case I, we number the transient states 1, 2, ..., m. Define the random variable $X_{ij}(t)$ as equal to one if the engagement is in state j at time t, given start in state j at time 0, and define $X_{ij}(t)$ as zero otherwise. Then the total time in state j is simply:

$$\int_0^T X_{ij}(t)\, dt \quad .$$

The expected value of this time is:

$$\int_0^T E(X_{ij}(t))dt = \int_0^T \varphi_{ij}(t)\, dt$$

where $\varphi_{ij}(t)$ is the probability that the engagement is in state j at time t, given start in state i at time 0. The quantities $\varphi_{ij}(t)$ are the state transition probabilities for the semi-Markov process and are calculated with the methodology of reference 11.

With this methodology, the engagement domination index (expected fraction of time in state O or higher) can be calculated. However, the old process, with all seven states, must be used so that access to all the states is possible. The new process contains only the two transient states O and D, so that there is no way to calculate the expected fraction of time in, say, OW.

APPENDIX B

FIRING SEQUENCE MODEL METHODOLOGY

APPENDIX B

FIRING SEQUENCE MODEL METHODOLOGY

This appendix consists of two parts. In part I, the formula for $f(e|t)$ is derived. In part II, a listing of sequences of firings obtained by Monte Carlo simulation is presented. This listing simulates a data base and is used to illustrate the Firing Sequence Model methodology computations presented in the text.

PART I - FORMULA FOR $f(e|t)$

The main text explains that the frequency function $f(e)$ provides the proper weighting for $p_W(e)$ for computing the overall probability of a win by a combatant, where $f(e)$ is the probability that an engagement produces the particular sequence $e = (s_1, \ldots, s_k)$ for arbitrary k. The "obvious" way to estimate $f(e)$ for any given e is to collect engagement data, count the number of occurrences of this e, and compute the fraction of times that this e occurred. This technique will certainly produce a better estimate of the probability of a win than obtained by weighting each $p_W(e)$ identically. However, if the data base contains only two or three hundred points, $f(e)$ for the more common sequences will be estimated with fair accuracy, although the less common sequences may have estimation errors of several hundred percent. Precise estimation for the less common sequences may require a data base exceeding 10,000, in which case $f(e)$ for the common sequences is estimated far more accurately than may be required. It is desirable to have an estimation procedure that provides more uniform accuracy of estimation.

As shown in the main text, we can estimate $f(e)$ by estimating $f(e|t)$ and $h(t)$, where $h(t)$ is the probability that an engagement lasts t units of time if all firings fail to kill. This latter term may be estimated from the range data or by tactical considerations related to a specific combat scenario. It remains to estimate $f(e|t)$ and show that it is advantageous to do so.

Before proceeding with the development, some additional symbology is required. For $s = 0, 1, 2$, let $F_s(.)$ denote the conditional distribution of time to first engagement firing given the firing is of type s. For each pair $s_1, s_2 = 0, 1, 2$, let $F_{s_1, s_2}(.)$ denote the conditional distribution of the time to a firing of type s_2, given the previous firing is of type s_1. All other notation is defined in the main text.

Suppose that $e = (s_1, s_2)$, where $s_i = 0, 1, 2$, for $i = 1, 2$, and the engagement is terminated at time t. The probability of observing e is $p(s_1)\, p(s_2 | s_1)$ multiplied

by the probability that there are only two firings in $[0, t]$. The probability of two or more firings in time t is the probability that the time to the second firing is less than or equal to t. This is therefore given by $F_{s_1} * F_{s_1, s_2}(t)$, i.e., the convolution of the distribution of the time to the first firing (of type s_1) and the distribution of the time between the first and second firings (of type 2). Similarly, the probability of three or more firings in time t is given by $F_{s_1} * F_{s_1, s_2} * H_{s_2}(t)$, where H_{s_2} is the distribution of the time between the second and third firings, unconditional on the type of the third firing. The function $H_{s_2}(t)$ is given by the relation:

$$H_{s_2}(t) = \sum_{s_3} p(s_3 | s_2) F_{s_2, s_3}(t)$$

where the sum is over the index $s_3 = 0, 1,$ and 2. Hence, $f(e|t)$, for this case, is given by:

$$f(e|t) = g(e|t)\, p(s_1)\, p(s_2 | s_1)$$

with
$$g(e|t) = \left[F_{s_1} * F_{s_1, s_2}(t)\right] - \left[F_{s_1} * F_{s_1, s_2} * H_{s_2}(t)\right] .$$

In the general case, $e = (s_1, \ldots, s_k)$. The probability of observing e is the product of:

$$p(s_1) \prod_{i=2}^{k} p(s_i | s_{i-1})$$

and the probability of precisely k firings in the time interval $[0, t]$. The probability of k or more firings in time t is the probability that the time to the kth firing is less than or equal to t. This is therefore given by the $(k-1)$ fold convolution:

$$G(t) = F_{s_1} * F_{s_1, s_2} \ldots * F_{s_{k-1}, s_k}(t) .$$

Similarly, the probability of $k+1$ or more firings in time t is given by $G * H_{s_k}(t)$,

where H_{s_k} is given by:

$$H_{s_k}(t) = \sum_{s_{k+1}} p(s_{k+1} | s_k) F_{s_k, s_{k+1}}(t)$$

where the sum is over the index $s_{k+1} = 0, 1,$ and 2. Hence, $f(e|t)$, for the general case, is given by:

$$f(e|t) = g(e|t)\, p(s_1) \prod_{i=2}^{k} p(s_i | s_{i-1})$$

with

$$g(e|t) = G(t) - G^{*}H_{s_k}(t) .$$

In the most general case, there are three probabilities $p(s_i)$, nine probabilities $p(s_i | s_j)$, and the corresponding distribution functions. These quantities are estimated using whatever data is available (presumably those representing the more common sequences). However, with estimates of these quantities in hand, the probabilities $f(e|t)$ for all the less common sequences can be calculated as above.

The carrying out of this procedure for the general case is still an ambitious procedure. Fortunately, however, it does not appear that the full methodology is required for many kinds of ACM analysis. First, if there are no firings of type 2 (no simultaneous firings), then $i, j = 0, 1$ and there are two probabilities $p(s_i)$, four probabilities $p(s_i | s_j)$, and associated distribution functions. The general case requires estimation of 24 quantities, while this case requires only 12. In some cases, it may be justified to assume that some of the distributions are equal, with a corresponding reduction in the need for estimation.

Second, unless the analyst is working with a priori distributions, it may not be necessary to perform the convolutions indicated above. For example, if $e = (s_1, s_2)$, $g(e|t)$ is just the probability of precisely the two firings s_1, s_2 in $[0, t]$. If the data base contains sequences whose first two firings are precisely s_1, s_2, then $g(e|t)$ can be estimated directly from the data.

PART II - LISTING OF FIRING SEQUENCES

Table B-1 lists a family of firing sequences resulting from a simulation of 200 ACM engagements. The data was produced by using a Monte Carlo simulation of the Maneuver Conversion Model to generate the sample engagements using the parameters displayed in the text. The engagements were automatically terminated after 3 minutes, with the combatants given an initial load-out of four air-to-air missiles each. This engagement sample was used to generate the parameter values used in the text to illustrate the Firing Sequence Model.

In table B-1, the entry "1" indicates a simulated firing for the evaluation fighter, and "0" indicates a simulated firing for the opposition fighter. Thus, the entry "1, 0, 1" indicates the occurrence of a 3-minute engagement in which the evaluation fighter fired first, the opponent second, and the evaluation fighter third and last. A dash (–) denotes an engagement in which no shots were recorded.

TABLE B-1

LIST OF 200 SIMULATED FIRING SEQUENCES

0	–	1,1	0
1	0	–	1,0,1
1	1	0	–
1	1,1	1,1	1,1
1	1	1	–
1,1	–	–	1,1
0	1	1	–
1	–	0,0	1,1
–	1	1,1,1	–
1,1	1	1	1
1,1	0,0	1	1
0,0	0	1	–
0,0	1	1,0	1
0	–	–	0
–	1	0	–
–	1,1	–	1
0	1	1,1	0,0
0,0	1	1,10	–
1,1	1,1	–	0
–	–	1	–
0,1	1,1	1,0	1
0,1	1,1	1	1

TABLE B-1 (Cont'd)

–	–	–	0, 0
1	–	–	1, 1, 1, 1
1	1, 1	1	1
1, 0, 0	1	1, 1	1, 1
1	–	1	0
1	1, 1	1	1
1, 0	0, 0	1	–
–	1, 1	0	1, 0, 0
1, 1, 1, 1	1	0, 1	1, 1
1	1	0	1, 1
–	–	–	–
1	–	–	–
1	1	1	1, 1, 1
1	0	1, 0, 0	
–	–	1	
–	0	1	
1, 1, 0	0, 0, 0	–	
0	–	–	
–	–	0, 1	
–	0, 1	0, 1, 1	
0, 1	–	0	
1, 1	1, 1	1, 1, 0, 1	
1	0, 1, 0	1, 0	
–	–	0	
–	1, 1, 0	1, 1	
1	–	1	
1	1	–	

TABLE B-1 (Cont'd)

1, 1	1	1
1, 1	–	1
1	–	–
1, 1	0	–
1	0, 0	–
0, 0	0	–